Superconducting Josephson Junctions:  
A Demonstration of the AC Josephson Effect and the Meissner Effect in Low-Temperature Niobium

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We explore the properties of a superconducting Josephson junctions manufactured from high-purity Niobium, at temperatures between 4.2 K and 9.25 K (the critical temperature for the material). A method for determining the width of junctions on an unknown junction chip (when the possible design widths are known) is presented, and applied to our particular set of junctions. This information is then used to demonstrate that the presence of an external magnetic field modulates the critical current $I_C$ in a Fraunhofer diffraction pattern, and that the minima of this pattern can be used to measure the London penetration depth. For Niobium at 4.2 K, we measure this value to be $\lambda = 38.8(5)$nm. Additionally, we explore the behavior of the junctions as temperature increases and investigate some discrepancies which arise between the theoretical predictions and the data collected.

I. BACKGROUND:  
THEORY OF LOW-TEMPERATURE SUPERCONDUCTING JUNCTIONS

A. Superconductivity

Low-temperature superconductivity results from the condensation of electrons in a metal into a lower-energy state called a Cooper pair. When the energy difference between these pairs and the state of the electrons at high-temperature (commonly termed $\Delta$) is sufficiently large, then the low-energy interactions such as scattering off of other electrons which are normally responsible for electrical resistance in a metal becomes forbidden, with the result that a current can flow freely through the substance in question. This purely quantum property is of course of great practical interest but also provides a valuable area for theoretical explorations.

The superconductive phenomenon appears only at temperatures below a critical level $T_C$ (which depends upon the material), at which point thermal excitations are insufficient to break apart all of the Cooper pairs. As the temperature is lowered below this point, more Cooper pairs appear and the amount of current which can be carried by the superconductor will increase.

1. BCS Model

While a naive treatment of a superconductor as a two-fluid system consisting of regular electrons and superconducting charge-carriers is accurate to a good approximation, the current best description of the complete phenomenon is the BCS mode, which postulates a small effective attraction between the charge carriers in a metal (smaller, of course, than their Coulomb repulsion) which allows the Bosonic Cooper pair states to form among electrons near the Fermi surface.

A prediction of this model which will be of particular interest in this report is a phenomenon called the Meissner Effect, in which a magnetic field applied to a superconductor is seen to be excluded from the interior of the superconductor below a certain threshold depth $\lambda$, called the London penetration depth. This behavior is the result of the appearance of currents within the superconductor (flowing indefinitely in the absence of resistance) which create their own magnetic field directly canceling the one which was applied. Within the London penetration region there are not enough charge carriers to completely cancel the field with supercurrents, so the field falls off exponentially as the number of carriers increases.

B. Josephson Junctions

The superconductors we study in this experiment are arranged into Josephson Junctions, with two layers of superconductive material on either side of an insulating barrier. Because of quantum mechanical tunneling, when a phase difference exists between the wave function of the charge carriers on each side a small current can flow through the classically insulating material even in the absence of an applied voltage. Of course, when enough current is applied to the junction, electrons become capable of crossing the energy gap, but the result is the appearance of a voltage drop across the junction (much like the drop across a diode). The magnitude of the current which causes this phenomenon to appear is known as the critical current, $I_C$, and the increase in the voltage at this point is naturally of magnitude $2\Delta$. The current can be further increased at this point without a significant increase in the voltage up to a current known
as $I_{\text{step}}$; beyond this level the superconductor begins to show ohmic behavior once again.

The fundamental theory governing these voltages and currents can be summarized by the Josephson equations:

$$I = I_C \sin \gamma, \quad (1)$$

$$V = \frac{\Phi_0}{2\pi} \dot{\gamma}, \quad (2)$$

in which $\gamma$ is the phase difference between the wavefunctions on either side of the insulator and the symbol $\Phi_0$ represents the quantum of magnetic flux through the superconductor and is known to be given by $h/2e$ (the inverse of this is the Josephson constant). These equations give rise to the properties outlined above, which can be seen experimentally in an I-V curve of the junction, formed by sweeping out a range of applied currents and measuring the resulting voltage drop across the junction. This curve will resemble a diode curve in some respects; however, the Josephson equations predict some hysteresis, which allows the current to fall back all the way to zero before the voltage drop vanishes once it has first been swept up past the value of $I_C$ (see figure 1). A full treatment of this phenomenon can be found in [1] and [2].

1. **Meissner Effect in a Josephson Junction**

Within a Josephson junction, the supercurrents produced in the presence of a magnetic field by the Meissner effect will interfere with tunneling in the junction in such a way that critical current is diminished and, indeed, periodically vanishes. The predicted nature of this interference is actually mathematically identical to the interference of a wave in a two-slit experiment, and thus produces the well-known Fraunhofer equation when the critical current is plotted as a function of the applied magnetic field. Mathematically speaking this means that, for a magnetic flux $\Phi$ across the junction, we have

$$I_C(\Phi) = I_C(0) \left| \frac{\sin \frac{\Phi}{\Phi_0}}{\frac{\Phi}{\Phi_0}} \right|. \quad (3)$$

2. **Temperature Effects**

All of the foregoing effects are, naturally, dependent on the temperature of the junction at least indirectly. The most fundamental of these dependencies are in the critical current and size of the energy gap $\Delta$, since changes in these quantities trickle down into everything else. The changes in these quantities as temperature increases can be understood heuristically as the result of more thermal energy being available to break apart the Cooper pairs, which in turn reduces the value of the critical current. The size of the band gap $\Delta$ also depends upon this, because the breaking of a Cooper pair requires a change in the energy of all other pairs in the semiconductor.

The theoretical dependencies behind these relationships are far more complex. The critical current, for example, depends explicitly upon the temperature and also implicitly through it’s dependence on the band gap size:

$$I_C(T, \Delta(T)) = \frac{\pi \Delta(T)}{2eR_n} \tanh \frac{\Delta(T)}{2k_B T}. \quad (4)$$

In this equation, $R_n$ is the resistance of the superconductive material in its normal phase.

The temperature dependence of the gap size itself is even more complicated, and can only be described through means of an integral equation. However, a very good empirical fit to the solution has is shown in [1] to be given by

$$\left( \frac{\Delta(T)}{\Delta(0)} \right)^2 = \cos \left[ \frac{\pi}{2} \left( \frac{T}{T_C} \right)^2 \right]. \quad (5)$$

**II. EXPERIMENTAL TECHNIQUE**

Our experiment consists of an enclosed probe (containing the array of Josephson junctions and other components) and a set of measurement devices capable of interfacing with the probe and recording data from it. The details of this equipment are outlined below, as are the techniques we use as cross-checks to verify the reliability of our data.
A. Apparatus

1. Junctions and Probe

At the heart of the experiment is our probe apparatus, which houses the Josephson junctions themselves. The junctions, which consist of niobium superconductors separated by an aluminum oxide layer, were produced at Stony Brook university using a fast fabrication process which should minimize subgap resistance and fluctuations in the value of the critical currents [3]. The chip embedded inside the probe contains many such junctions, four of which are connected directly to external leads in the probe which allow us to measure their properties directly by applying a current (a fifth pair of leads coming from the junction chip is available in the probe output, but as will be shown below, experimental evidence suggests that these connections were accidentally wired so that they cross several junctions connected in series instead of a single junction). A switch at the probe interface allows us to select which of the junction’s terminals are connected to our measurement devices at any particular time, and a second switch is used to short the junctions out of the circuit whenever adjustments are being made which might cause a current surge and damage the fragile superconductors.

Placed immediately atop the junction chip inside the probe is a Germanium thermistor, which we use to indirectly measure the temperature inside the probe. As an intrinsic semiconductor, Germanium’s resistance increases as the temperature is lowered, since less thermal energy is available to assist the electrons in reaching the conduction band. This particular thermistor has a well-documented calibration curve which has been empirically fit to high-precision over its range of operation. The fit used to compute temperatures in our experiment is documented in Appendix B. Leads connected directly to this thermistor are also available at the probe interface, allowing us to monitor it’s resistance (and hence the temperature of the probe) as the equipment is cooled or heated.

The final component of the probe is a small resistor located adjacent to the junction chip and thermistor, which is used as a heater. When it is necessary to raise the temperature inside the probe, a voltage is applied directly across this resistor via the probe interface, and the resulting power dissipation is used to heat up the system. The resistance of this heat element is approximately 1KΩ, although its exact value is not necessary for the experiment. These components are collectively enclosed inside a vacuum can, which is used (in vacuum state) to insulate the junction from outside influences, and can also be filled with exchange gas as needed when the temperature must be adjusted. The vacuum can is also surrounded by coils of wire (conductive even at low temperatures), positioned so that they can be used to generate a nearly uniform magnetic field within the junctions. Magnitude of the field has been previously calibrated to be directly proportional to the current in the coils, with a constant of proportionality of 0.25(2)G/mA.

2. Probe Input Apparatus

In order to control the conditions inside the probe, we need to be able to apply standard currents and voltages through the probe interface. For example, in order to measure the I-V characteristics of the junctions, we will apply a current swept across a range of positive and negative values. To accomplish this, a WaveTek voltage source is employed to sweep voltage triangle waves, which are then applied across a standard resistor (measured to have resistance 998.6(1)Ω). The resulting current then flows through the probe and the Josephson junctions before returning to the WaveTek.

We control the current in the coils (used to generate the magnetic field) in a similar way; however, in this case, only constant currents are desired, and so a series of DC voltage supplies are used to apply a controlled voltage to a General Radio decade resistor. The same procedure is also used to supply a current to the heating element, although in this case, to maximize the current (and hence the power dissipated) we use a minimal setting of 10.0(1)Ω on the decade resistor. All these currents are applied through appropriate connections on the probe interface.

B. Measurement Techniques

Signals from the probe are communicated via a cable from the probe interface to our workspace, where they can be sent to the appropriate measurement device as needed. In particular, two HP DMM’s are used to read voltage and create the I-V curves: one to measure the voltage across the selected junction, and one which indirectly measures the current applied to the junction by measuring the voltage drop across the 998.60(5)Ω input resistor described above. Since each of these signals is extremely small, we also employ two amplifier modules to boost the signals before they are measured by the DMMs. These modules contain an adjustable frequency filter (used to suppress high-frequency noise) and a standard op-amp. The gain in the amplifier used to measure the junction voltage is measured to be \(G_1 = 76.9(5)\), and the gain in the voltage signal across the input resistor is \(G_2 = 76.2(5)\).

As described above, the temperature in the probe is measured indirectly by observing the resistance of the Ge thermistor. To do this, we use a Keithly DMM to measure the voltage across a General Resistance decade resistor, set at 29990Ω and measured to be 29990.0(1)Ω. (It is necessary to keep this resistance high in order to prevent excessive power dissipation within the thermistor itself, which would cause self-heating and misleading...
temperature measurements). This voltage measurement, combined with the known resistance, tells us the current through the thermistor, so a second Keithly is employed to measure the voltage across the thermistor itself. The resistance of the germanium is therefore given by

$$R_T = \frac{V_{\text{thermistor}}}{V_{\text{decade}} 299902}$$

and it is this value which is used to calculate the temperature as described in Appendix B.

When temperature does not need to be measured, one of the Keithly DMM’s is repurposed to measure the current through the magnetic field coil. This, too, is done indirectly, by measuring the voltage drop across the General Radio decade resistor which is used to control the current.

With the exception of data collected to measure temperature, which is read off by hand once the system has reached thermal equilibrium, the data from the DMM’s described above are recorded by a LabView program created specifically for the experiment. Except where noted, we take data with a fixed X and Y range (to reduce delays) and a DMM integration time of 0.1. Note that all such data taken in this way must be adjusted to reflect the fact that the DMM’s are measuring amplified voltage signals, and not the actual voltages and currents present in the probe. A digital HP Oscilloscope is also used to allow us to view the signal behavior before taking data with LabView.

Critical currents, in particular, are detected using LabView’s “Trigger” setting. By setting the trigger threshold to its minimum value of 0.1V and stopping the data taking immediately after the critical current is reached, LabView will record the current when the voltage across the junction suddenly increased upwards past the trigger threshold, giving us the positive critical current for the junction. Alternatively, a trigger level of −0.1V can be used to detect the critical value when the current is applied in the opposite direction. The size of the band gap, 2Δ, can also be measured from this data, but must be estimated by hand by reading off the nearest data point instead of measured with a trigger. The resulting uncertainty is considered as part of the overall uncertainty of the experiment.

1. Verification of Measurement Apparatus

To confirm that these measurement equipment are all functioning as intended, we first test them on an ersatz probe module, which contains a resistor of known value as a stand in for the junction (as well as a diode network which emulates the junction behavior) and an additional resistor which can be used to test our technique for measuring the resistance of the probe thermistor. Full details of this test procedure can be seen in our mini-lab report, included here as Appendix C). The results of these initial tests showed that we can successfully measure the I-V curves of the junctions in the probe as well as the resistance of the thermistor (and therefore the temperature inside the probe). The precision of these measurements appears to be principally limited by the difficulty in measuring directly the gain of the amplifier modules, which introduces the largest source of uncertainty. Note, however, that this uncertainty does not effect our measurements of the thermistor resistance, which appear to be highly precise and limited only by small random fluctuations in the temperature due to thermal noise.

C. Experimental Procedure

The equipment described above are used to take data in three phases: at constant low temperature and no applied magnetic field, at low temperature while varying the field strength, and then at a range of higher temperatures with no field applied. In preparation for the first of these phases, the probe is sealed in the vacuum can, which is filled with gaseous nitrogen to facilitate the exchange of heat. The vacuum canister is then immersed in liquid nitrogen and and cooled to around 77K. After a check to ensure the electrical connections have survived the cooling process, the nitrogen in the vacuum can is replaced with gaseous helium and the canister is inserted into the dewar of liquid helium. We lower the helium gradually into the dewar (to keep the pressure of vaporized helium at an acceptable level) and let it cool for approximately one hour, until the temperature appears stable at 4.2 K. At this point, we lower the probe the rest of the way into the dewar and pump out the exchange gas to ensure that the probe stays at low temperature.

While fully cooled, we measure the I-V curves of each of the five junctions, taking data as outlined above. The applied currents are created from triangular voltage waves with a frequency of 0.1 HZ. These voltages are applied across the 1K input resistor, resulting in currents which range in general from about -100 to 100 µA. From the resulting I-V curves we detect the critical current of each junction and the value of 2Δ, as well as the value of I_{step}, the total amount by which the current can be increased while the junction voltage remains at a voltage of 2Δ. These data will allow us to infer the dimension of each of the five junctions in our chip.

In the subsequent phases of the experiment, we focus our attention only on junction 5, which we judge to be the most “well-behaved” of the junctions. We begin by applying controlled magnetic fields to the junction, which is done by increasing the current in the coils in somewhat arbitrary steps and using LabView to record the critical current at each value of the field. Since we are ultimately interested in determining the penetration depth, which requires a knowledge of the magnitude of the field which causes the critical current to vanish, we continue increasing the field until we have seen two “periods” of the ex-
pected Fraunhofer pattern emerge in the results. This data will thus give us two different nodes from which to estimate the penetration depth. In the end, we find that this requires an applied voltage of between 0 and 5 V, which corresponds to a current ranging between 0 and 0.5 mA. Periodically during this procedure we also check that the temperature of the probe remains at 4.2 K, in case the small resistance of the coil has caused any heat to be generated.

Once these low-temperature measurements have been completed, we raise the probe above the surface of the remaining helium in the dewar and pump gaseous helium back into the canister. The heater is then used to gradually raise the temperature of the canister, and for each value of $T$ we measure the critical current and voltage gap of junction 5 in LabView. As with the magnetic field, there is no need to proceed in exactly-sized intervals; instead, we increase the voltage across the heating resistor by some standard amount (usually about 2 V) and wait for the new probe system to reach thermal equilibrium. We then calculate the temperature as described above and continue to increase the voltage if the new temperature is not sufficiently larger than at our previous data point. In general, we aim to take data approximately every 0.4 K, over a range from 4.2 to 9.7 K, which in practice requires applied voltages between 0 and 68 V. We also attempt to detect the critical temperature where the niobium goes normal during this procedure, which completes our data taking.

III. RESULTS

A. General Junction Behavior

1. Measurement of Characteristic Properties

From our low-temperature measurements (at $T = 4.2$ K) with no applied magnetic field, we measure the critical currents and band gap magnitudes of each junction. The results are summarized below in figure I. The primary component in the uncertainties remains the uncertainty in the gain of the amplifiers, although there is some inherent uncertainty in the accuracy of the data points are recorded by LabView and a small contribution from the uncertainty in the input resistor.

2. Determination of Junction Dimensions

From the values in table I (and a knowledge of the expected size of the junctions on the chip), we can determine the actual size of the junctions in practice. Since the Josephson equations prescribe the size of the current flux as a result of the superconductor properties, the total current simply scales with the cross sectional area. From [3] we know that these semiconductors were manufactured in such a way as to make them nearly perfect squares. Hence, for any junction $i$ we have

$$I_{\text{step},i} = \Phi_0 w^2$$

where $w$ is the width (and length) of the junction. Since each junction is made of identical materials, the flux at critical current will be identical, with the result that for two junctions $i$ and $j$ we have the ratio

$$\sqrt{\frac{I_{\text{step},i}}{I_{\text{step},j}}} = \frac{w_i}{w_j}.$$ 

Quick calculations thus allow us to determine the approximate ratio between the sizes of the junctions. In particular, $w_1/w_2 = 1.74 \approx 7/4$, $w_2/w_5 = 0.78 \approx 4/5$, and $w_4/w_1 = 0.97 \approx 1$.

Since we know the “design widths” of the junctions (the widths they were believed to have as manufactured) then we can identify each junction with the design width necessary to approximately satisfy the ratios above. These results are summarized below in table III.

In junction 4 we note a peculiarity: to satisfy the ratios, it must have a width of about 9 µm; however, according to the chip schematic, no junction of this size is hooked up to the probe output. A closer examination of the I-V curve for junction 4 (figure 2) reveals another unusual feature: instead of the expected instantaneous jump from $V = 0$ to $V = 2\Delta$ at $I_C$, the voltage lingers twice, making its way up in a stair-step pattern. Indeed, these stair-step points occur in intervals of the value of $2\Delta$ measured for all the other junctions. This leads us to suspect that this “junction” is actually a series of junctions inadvertently wired together inside the chip, producing a junction with an “effective” width of about 9 µm. Because the critical current and current step also closely resemble those of junction 1, we hypothesize that the two 7µm junctions are involved in this series.

As noted in [3], the design widths do not always match the actual widths of the manufactured junctions, but when they differ they do so by a constant offset. We can test for this point by plotting the square root of the current step against the width of the junction; if no correction is needed, the points should lie on a line passing through the origin. In this case, as can be seen in figure...
FIG. 2. IV characteristics curve for junction 4, taken at 4.2 K. The “stair-step” behavior which suggests that it is not a single junction, but three in series, can be seen plainly along the horizontal lines at $\pm I_C$.

3. The best-fit (produced by Mathematica using LinearModelFit) line has an $x$-intercept of $-0.013\mu m$.

FIG. 3. Plot of the square root of the $I_{step}$ value of each junction versus its design width. Note only junctions 1, 2, and 5 are plotted, since $I_{step}$ could not be measured for junction 3 and junction 4 is believed to consist of more than a single physical junction.

However, an examination of the confidence intervals for the parameters of the fit line shows that the intercept parameter is consistent with zero even at the 90% level. The junction sizes are thus taken to be as reported in Table II below, where the uncertainty in the actual width comes only from the fact that variations at this level would not be detected by the fit at this confidence level. As such, the uncertainty in the widths will be negligible in future calculations.

B. Magnetic Field Dependent Effects

With the probe temperature still stable at 4.2 K, we begin applying magnetic fields to measure the effect on the critical current. Since a high degree of symmetry was observed between the positive and negative values of critical currents during the preceding observations (in fact, the two were equal to within the margin of error) we now choose to record the positive critical current. We also restrict our focus to junction five, finding that it’s properties are best-suited to the range of currents we can comfortably sweep and measure.

A plot of the resulting data is given below in figure 4. As predicted by equation 3 above, the critical current periodically drops to zero and behaves sinusoidally in between these nodes.

1. Meissner Effect

Since the magnitude of a single quantum of magnetic flux is known essentially exactly to be $\Phi_0 = 2.07 \cdot 10^{-15}$ Wb [4], and the flux is equal to the applied field times the apparent cross-sectional area, then we have the equation

$$\Phi_0 = wt_{eff}(\lambda)B_0$$  \hspace{1cm} (9)

where here the effective thickness $t_{eff}$ represents the total thickness of the junction in which the magnetic field has not been expelled by the Meissner effect. This includes the entire thickness of the oxide layer in the center of the junction (known to be $5.0(1) \cdot nm$), and two slices of niobium outside the London penetration depth $\lambda$. Hence we have

$$\Phi_0 = w(2\lambda + 5.0nm)B_0$$  \hspace{1cm} (10)

Or, solved for $\lambda$,

<table>
<thead>
<tr>
<th>Junction</th>
<th>Design Width ($\mu m$)</th>
<th>Width ($\mu m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>7.00(1)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4.00(1)</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>9.00(1)</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>7.00(1)</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5.00(1)</td>
</tr>
</tbody>
</table>
The (positive) critical current $I_C$ of junction 5 as a function of the applied magnetic field, in Gauss. The features of a Fraunhofer diffraction pattern can be seen, including two complete periods.

$$\lambda = \frac{\Phi_0}{2\omega B_0} + 2.5 \text{nm} \quad (11)$$

The magnetic field required to make the critical current reach a minimum $(B_0)$ can be extracted from the data in figure 4 by fitting it with a generalized Fraunhofer diffraction function and locating the minima. However, due to idiosyncrasies of Mathematica’s NonlinearModelFit function, we find that rather than fit the data in the final form displayed above, it is easier to fit the data as they are originally read off the equipment: the voltage applied to create the current, $V_{IC}$, versus the voltage needed to create the magnetic, $V_B$. This will also eliminate uncertainty arising from uncertainty in the amplifier gain. Up to the first minimum, a fit of the form

$$I_C = \alpha \left[ \sin \pi \frac{V_B}{\beta} \right]$$

gives $\beta = 2.28(5) V$ and thus the first minimum occurs at $V_B = 2.28(5) V$. From this we can calculate the applied magnetic field at this point to be $B_0 = 57(3) G$ or $5.7(3) \cdot 10^{-4} T$. Plugging this into equation 11 gives $\lambda = 38.8(5) \text{nm}$, which is consistent with the value of 38(5) nm found by Maxfield and McClean [5]. The Maxfield and McClean value is calculated for 0 K; however, as they also show that the temperature dependence of the penetration depth will flatten as temperature decreases, we would still expect our result to be consistent.

C. Temperature Dependent Effects

1. Gap Voltage

After measuring the magnetic field dependencies of junction five, we begin to study its properties as the temperature is increased. In particular, we track the value of the gap voltage $2\Delta$ as the temperature increases. From this we can calculate the dimensionless parameters $t = T/T_C$ and $\delta = \Delta(T)/\Delta(0)$, which theoretically obey the (approximate) relationship

$$\delta(t) = \sqrt{\cos \frac{\pi t^2}{2}} \quad (13)$$

[1].

We take the value of $T_C$ to be 9.25K [6]. The value of $\Delta(0)$ for strong-coupling Niobium is approximated by the weak-coupling result from BCS theory, divided by a factor which accounts for a number of corrections, including the anisotropy of Niobium and the structure of the gap function. A complete treatment of these corrections is given in [7], with the final result that

$$\Delta(0) = \frac{1.76 K_f T_C}{1 - \exp(-2/N_0 V^*)}, \quad (14)$$

where the correction factor $N_0 V^*$ for Niobium has been estimated as 0.89. Our experimental values are plotted against the theoretical expectation in figure 6; the approximate shape and curvature agree, however, a significant offset exists between our values and the theoretical
curve. This suggests a systematic error in our procedure for extracting the values of $2\Delta$ from the IV curves recorded in LabView. With no rigorous procedure to identify the $2\Delta$ point, values were approximated by eye using a consistent but somewhat arbitrary convention. Had time permitted, the discrepancies between the measured and predicted values could perhaps have been used to refine this procedure.

Time did not permit an extensive study of this discrepancy, however, the data at lower temperatures and at higher temperatures were taken in two separate sessions, so it is possible that some conditions changed between the two.

![Figure 6](image1.png)

**FIG. 6.** Theory curve and experimental data points for the ratios $\delta$ and $t$, which are proportionate to the gap voltage and the temperature of the junction. The discrepancy likely represents systematic error in our procedure for measuring $2\Delta$.

2. **Critical Current**

The theoretical temperature dependence for the critical current (from [1]) is

$$I_C(T, \Delta(T)) = \frac{\pi \Delta(T)}{2e R_n} \tanh \frac{\Delta(T)}{2k_B T}$$

(15)

We measure $R_n$, the normal resistance of the junction, by fitting the IV curve of the junction for a voltage much larger than $2\Delta$ and calculating the reciprocal of the slope. This gives $R_n = 74.0(7)\Omega$. Using this values and our measured values of $\Delta(T)$, we can compare the relationship between the two to the expected theoretical result. The theoretical curve is plotted against our measured data in figure 7. The two agree well as temperatures approach $T_C = 9.25$. We present a procedure for identifying the widths of the junctions on our chips by measuring $I_C$, $I_{step}$, and $2\Delta$ for each junction. Next, measuring the effect of the application of a magnetic field on $I_C$, we show that the relationship between the two produces a Fraunhofer diffraction pattern, which we can then use (in concert with our newly-gained knowledge of the widths) to calculate the penetration depth for niobium at 4.2 K to be $\lambda = 38.8(5)nm$, a figure which is highly precise and consistent with other published results. We then study the behavior of our junction as we increase the ambient temperature. The change in the voltage gap displays the expected qualitative relationship, but is skewed by a constant offset factor. This likely represents a systematic error in our data-taking procedure, as there was no precise way to extract the values of $\Delta$ from the data we collected. We also see some discrepancy in the temperature dependence of the critical current, which cannot be so easily explained. The most probable cause is a change in ambient effects (such as external magnetic fields) during the course of data collection.

![Figure 7](image2.png)

**FIG. 7.** Theory points and experimental points showing the critical current of junction 5 as a function of temperature. Note that the theory points still depend on experimentally measured values of $\Delta$ and are only intended to show the relationship that would be expected in our data between the critical currents at different temperatures, not predicted values of the current at those temperatures.

**IV. CONCLUSIONS**

Our results demonstrate the expected behavior of a superconducting Josephson junction at low temperature, while displaying some inconsistencies in the temperature dependence as the temperatures approach $T_C = 9.25$. We present a procedure for identifying the widths of the junctions on our chips by measuring $I_C$, $I_{step}$, and $2\Delta$ for each junction. Next, measuring the effect of the application of a magnetic field on $I_C$, we show that the relationship between the two produces a Fraunhofer diffraction pattern, which we can then use (in concert with our newly-gained knowledge of the widths) to calculate the penetration depth for niobium at 4.2 K to be $\lambda = 38.8(5)nm$, a figure which is highly precise and consistent with other published results. We then study the behavior of our junction as we increase the ambient temperature. The change in the voltage gap displays the expected qualitative relationship, but is skewed by a constant offset factor. This likely represents a systematic error in our data-taking procedure, as there was no precise way to extract the values of $\Delta$ from the data we collected. We also see some discrepancy in the temperature dependence of the critical current, which cannot be so easily explained. The most probable cause is a change in ambient effects (such as external magnetic fields) during the course of data collection.
Appendix A: Ge Thermistor Calibration Curve

From the documentation which accompanied our thermistor, we procure the following empirical fits for the calibration curves which we use to convert measured resistances into temperatures in Kelvin.

The temperature $T$ as a function of resistance $R$ is given as

$$T(R) = \sum_{j} a_j \cos \{ j \arccos [X(R)] \}$$  \hspace{1cm} (A1)

where

$$X(R) = \frac{\left( \log_{10} (R) - Z_L \right) - \left( Z_U - \log_{10} 1 \right) }{Z_U - Z_L}$$  \hspace{1cm} (A2)

and the constants $a_j$ and $Z_U, Z_L$ are given in the table below:

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value (1.40 to 6.37 K)</th>
<th>Value (6.37 to 27.4 K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>3.672058</td>
<td>15.271648</td>
</tr>
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Appendix B: Testing the Measurement Devices

In this appendix we demonstrate the accuracy and usage of the labview system which we will use in the future to record the IV curve of the Josephson junction and to measure its critical current. To do this demonstration, we use the ersatz probe in the lab set up as a stand in for the junction, and measure the resistance of a particular resistor within the device. A Labview program is used to take the data points.

1. Apparatus and Procedure

The ersatz probe studied in this experiment consists of a series of resistors and a diode loop, with terminals between each so that the voltage across an individual resistor can be measured. The schematic for the probe, as provided in the lab writeup, is given below:

Using a Wavetek function generator, we sweep out a range of voltages across a 1-KΩ resistor, producing a current which can be known with high precision by measuring the voltage drop. This current runs through the selected resistor (in our case, resistor $R_2$) in the ersatz probe, across which the voltage drop is also measured. Both voltage signals are amplified using nearly identical amplifier boxes with no signal filtering. The amplified signals are plotted against each other using Labview to produce a V-V scatter plot. Then, since the value of the initial resistor is known, we can calculate the resistance of $R_2$ from the slope of the best-fit line, which is calculated in Excel. The complete schematic is given below in figure 8.

![FIG. 8. Schematic of the ersatz probe](image-url)
2. Results and Error Analysis

Since we ultimately need to know the current through $R_2$, it is first necessary to know the actual resistance of the so-called 1K resistor to which the Wavetek is connected. A direct, active measurement of the effective resistance (including resistance from diaxial cables, connections, etc.) was made using the DMM, giving the value $R_{1K} = 998.60(05)\,\Omega$. Combining this information with the measured values of the gains given above, we can find an equation for $R_2$ as follows:

$$R_2 = \frac{V_{\text{probe}}}{I_{\text{in}}} = \frac{R_{1K} V_{\text{probe(measured)}}}{V_{\text{in(measured)}}} = \left(\frac{G_{V_{\text{in}}} R_{1K}}{G_{V_{\text{probe}}}}\right)(-m^{-1})$$  \hspace{1cm} (B1)

Here, $V_{\text{measured}}$ is the voltage after amplification, as seen by labview, and $m$ is the slope of best-fit line produced from the data. Note that the minus sign is introduced simply because the terminals we measure are reversed in the probe apparatus from the input pins.

After taking several thousand data points with labview, we plot the data in Microsoft Excel and produce a linear best-fit curve with slope $m = -1.0970(5)$ and an $R^2$ value of 1 to within the error of Excel’s statistical function. The IV curve is thus very reliably linear, as expected, and as a result we expect only negligible statistical errors resulting from the fit. Instead, the only contribution is truncation error in the slope parameter, since Excel’s rounding criterion is unknown.

Using this value of $m$, we can use Eqn. 1 to calculate $R_2$:

$$R_2 = \left(\frac{G_{V_{\text{in}}} R_{1K}}{G_{V_{\text{probe}}}}\right)(-m^{-1}) = \left(\frac{76.2 \cdot 998.6\,\Omega}{76.9}\right)\left(\frac{1}{1.097}\right) = 902.0\,\Omega$$

Since we have identified uncertainties in each of the parameters in the equation, we can also estimate the uncertainty in our measurement by simply combining these errors in the usual way. In particular, since all the terms are combined multiplicatively, the relative uncertainties simply add in quadrature:

$$\delta R_2 = R_2 \sqrt{\left(\frac{\delta G_{V_{\text{in}}}}{G_{V_{\text{in}}}}\right)^2 + \left(\frac{\delta G_{V_{\text{probe}}}}{G_{V_{\text{probe}}}}\right)^2 + \left(\frac{\delta R_{1K}}{R_{1K}}\right)^2 + \left(\frac{\delta m}{m}\right)^2} = 8.3\,\Omega$$

3. Conclusions

Hence, our final reported value is $R_2 = 9.020(83) \cdot 10^2\,\Omega$. A direct measurement of the resistor, made by connecting the DMM directly to the appropriate pins of the ersatz probe box, gives $R_2 = 910.30(05)\,\Omega$, which is consistent with our more indirect measurement (if only barely so). Our measurement is also consistent with the value of $R_2 = 910\,\Omega$ reported in the schematic in figure 8. By far the largest contribution to our uncertainty is the uncertainty in the gain from the amplifiers, which was primarily caused by noise in the oscilloscope. In the future, a more detailed measurement of the gain might be warranted in order to reduce the overall uncertainty. This might be accomplished by using a slightly stronger initial signal or by averaging over measurements at a range of initial amplitudes.